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THE ELASTOPLASTIC PROBLEM OF THE HALF-SPACE WITH A HOLE SUBJECTED TO AXIALLY SYMMETRIC LOADING

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Abstract. The elastoplastic problem of the half-space with a hole subjected to axially symmetric loading considered in this paper is based on the elastoplastic deformation process theory. Solution of this problem is carried out by using the modified elastic solution method and the finite element method. Some results of numerical calculation are presented here to give the picture of plastic domains enlarging in the body and the obtained displacements on the free boundary of the half-space.

1. GOVERNING EQUATIONS

Let's consider an elastoplastic half-space with a hole subjected to axially symmetric loading, the strain state of which is determined by Cauchy relation

$$\varepsilon_r = \frac{\partial u}{\partial r}; \quad \varepsilon_z = \frac{\partial v}{\partial z}; \quad \varepsilon_\theta = \frac{u}{r};$$

$$\gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}, \quad \gamma_{r\theta} = \gamma_{z\theta} = 0;$$

where $u = u_r(r,z)$, $v = u_z(r,z)$ and $u_\theta = 0$ are displacement components.
The stress-strain relation for elastic state can be expressed as follows:

$$\{\sigma\} = \begin{bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \tau_{rz} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 \\ 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{bmatrix} = [D]\{\varepsilon\}, \tag{1.1}$$

where $[D]$ -the matrix of elastic constants

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = G = \frac{E}{2(1+\nu)}.$$

Stress and strain intensity are determined in the form

$$\sigma_u = \sqrt{\frac{3}{2} S_{ij} S_{ij}} = \frac{\sqrt{2}}{2} \sqrt{(\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2 + (\sigma_\theta - \sigma_r)^2 + 6\tau_{rz}^2}\,, \tag{1.2}$$

$$\varepsilon_u = \sqrt{\frac{2}{3} e_{ij} e_{ij}} = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_r - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_\theta)^2 + (\varepsilon_\theta - \varepsilon_r)^2 + (3/2)\gamma_{rz}^2}\,, \tag{1.3}$$

and the arc-length of the strain trajectory

$$s = \int_0^t \sqrt{\frac{2}{3}\dot{e}_{ij}\dot{e}_{ij}}\, dt. \tag{1.4}$$

For a plastic problem we use the elastoplastic deformation process theory [1]

$$d\sigma_{ijkl} = D_{ijkl} d\varepsilon_{kl}, \tag{1.5}$$

or in matrix form

$$\{d\sigma\} = \begin{bmatrix} d\sigma_r \\ d\sigma_z \\ d\sigma_\theta \\ d\tau_{rz} \end{bmatrix} = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 \\ D_5 & D_6 & D_7 & D_8 \\ D_9 & D_{10} & D_{11} & D_{12} \\ D_{13} & D_{14} & D_{15} & D_{16} \end{bmatrix} \begin{bmatrix} d\varepsilon_r \\ d\varepsilon_z \\ d\varepsilon_\theta \\ d\gamma_{rz} \end{bmatrix} = [D]_d\{d\varepsilon\}, \tag{1.6}$$

where $[D]_d$ is matrix of stress-strain relation of elastic-plastic behaviour, D_i can be written as following

$$\begin{aligned}
&D_1 = \lambda + 2G - H_1; \quad D_2 = \lambda - H_2; \quad D_3 = \lambda - H_3; \quad D_4 = -H_4, \\
&D_5 = D_2; \quad D_6 = \lambda + 2G - H_6; \quad D_7 = \lambda - H_7; \quad D_8 = -H_8, \\
&D_9 = D_3; \quad D_10 = D_7; \quad D_{11} = \lambda + 2G - H_{11}; \quad D_{12} = -H_{12}, \\
&D_{13} = D_4; \quad D_{14} = D_8; \quad D_{15} = D_{12}; \quad D_{16} = G - H_{16}, \\
&H_1 = \frac{4}{3}G\omega_1 + 3G(\omega_2 - \omega_1)\frac{S_{rr}^2}{\sigma_u^2}; \quad H_2 = -\frac{2}{3}G\omega_1 + 3G(\omega_2 - \omega_1)\frac{S_{rr}S_{zz}}{\sigma_u^2}, \\
&H_3 = -\frac{2}{3}G\omega_1 + 3G(\omega_2 - \omega_1)\frac{S_{rr}S_{\theta\theta}}{\sigma_u^2}; \quad H_4 = 3G(\omega_1 - \omega_2)\frac{S_{rr}S_{rz}}{\sigma_u^2}, \\
&H_6 = \frac{4}{3}G\omega_1 + 3G(\omega_2 - \omega_1)\frac{S_{zz}^2}{\sigma_u^2}; \quad H_7 = -\frac{2}{3}G\omega_1 + 3G(\omega_2 - \omega_1)\frac{S_{\theta\theta}^2}{\sigma_u^2}, \\
&H_8 = 3G(\omega_2 - \omega_1)\frac{S_{zz}S_{rz}}{\sigma_u^2}; \quad H_{11} = \frac{4}{3}G\omega_1 + 3G(\omega_2 - \omega_1)\frac{S_{\theta\theta}^2}{\sigma_u^2}, \\
&H_{12} = 3G(\omega_2 - \omega_1)\frac{S_{\theta\theta}S_{rz}}{\sigma_u^2}; \quad H_{16} = G\omega_1 + 3G(\omega_2 - \omega_1)\frac{S_{rz}^2}{\sigma_u^2},
\end{aligned} \tag{1.7}$$

where

$$\omega_1 = 1 - \frac{\sigma_u}{3Gs}; \qquad \omega_2 = 1 - \frac{\phi'(s)}{3G}.$$

If the body has elastic behaviour, then $\omega_1 = \omega_2 = 0$ and the matrix $[D]_d$ automatically reduces to $[D]$ in (1.1).

2. FINITE ELEMENT METHOD

The hole in half-space has cylindrical form with radius $R = R_0$ and height $H = 2R_0$ subjected to axially symmetric loading. The part of the meridian cross-cut half-space is discretized into 18 big triangle elements, 40 small triangle elements and 4 infinite elements, such as [3, 4], on this part of medium we put a system of cylindrical coordinates rOz (Fig. 1).

The finite element (e) having nodes (i, j, k) is studied. At a point $M(r, z)$ in the element (e) we choose:

$$\begin{aligned} u &= a_1 + a_2\, r + a_3\, z, \\ v &= a_4 + a_5\, r + a_6\, z \end{aligned} \tag{2.1}$$

and in the matrix form equations (2.1) can be written

$$\{u\}_{2\times1} = [F(r,z)]_{2\times6} \cdot \{a\}_{6\times1}. \tag{2.2}$$

In the 3 nodes (i, j, k) we have

$$\{q\}^e = \begin{bmatrix} {q_1}^e \\ {q_2}^e \\ {q_3}^e \\ {q_4}^e \\ {q_5}^e \\ {q_6}^e \end{bmatrix} = \begin{bmatrix} \{u\}^i \\ \{u\}^j \\ \{u\}^k \end{bmatrix} = \begin{bmatrix} 1 & r_i & z_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_i & z_i \\ 1 & x_j & y_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_j & z_j \\ 1 & r_k & z_k & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_k & z_k \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = [A]_{6\times6} \cdot \{a\}_{6\times1}. \tag{2.3}$$

Instead of finding $\{a_i\}$ we find displacement components $\{q\}^e$:

$$\{a\}_{6\times1} = [A]^{-1}_{6\times6} \cdot {\{q\}^e}_{6\times1}.$$

The displacement in a point M(r, z) is calculated through displacement of nodes (i, j, k).

$$\{u\}_{2\times1} = [F(r,z)]_{2\times6} \cdot [A]^{-1}_{6\times6} \cdot {\{q\}^e}_{6\times1} = [N(r,z)]_{2\times6} {\{q\}^e}_{6\times1}, \tag{2.4}$$

where

$$[N(r,z)] = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = [F(r,z)] \cdot [A]^{-1}$$

is matrix of functions form.

From (2.4) and Cauchy relation we obtain:

$$\{\varepsilon\}^e_{4\times1} = \begin{bmatrix} \varepsilon_r & \varepsilon_z & \varepsilon_\theta & \gamma_{rz} \end{bmatrix}^T = [B(q)^e]_{4\times6} \{q\}^e_{6\times1}. \tag{2.5}$$

For infinite elements (e) having two nodes (i, j) we can approximate displacements as follows

- elements 1

$$u = \frac{a_1}{r^2} + \frac{a_2}{r^2 z^2}; \quad v = \frac{a_3}{r^2} + \frac{a_4}{r^2 z^2},$$

- elements 2

$$u = \frac{a_1}{z^2} + \frac{a_2}{r^2 z^2}; \quad v = \frac{a_3}{z^2} + \frac{a_4}{r^2 z^2},$$

- elements 3

$$u = \frac{a_1}{r^2} + \frac{a_2 . z}{r^2}; \quad v = \frac{a_3}{r^2} + \frac{a_4 . z}{r^2}, \tag{2.6}$$

- elements 4

$$u = \frac{a_1.r}{z^2} + \frac{a_2}{z^2}; \quad v = \frac{a_3.r}{z^2} + \frac{a_4}{z^2}.$$

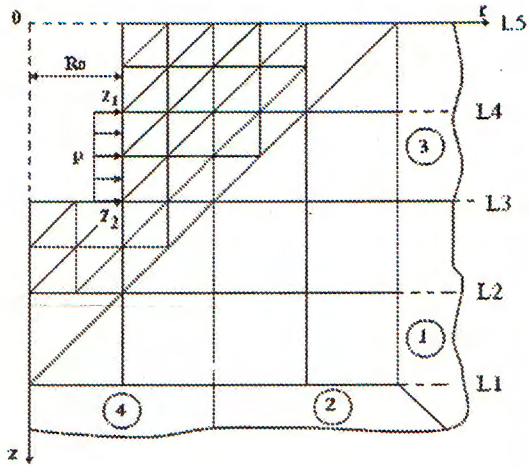

Fig. 1. The part of the meridian cross-cut half-space

Repeating caculation we give displacement in a point M(r, z) of infinite elements

$$\{u\}_{2\times1} = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}_{2\times4} \{q\}^e{}_{4\times1}, \tag{2.7}$$

from here

$$\{\varepsilon\}^e_{4\times1} = \begin{bmatrix} \varepsilon_r & \varepsilon_z & \varepsilon_\theta & \gamma_{rz} \end{bmatrix}^T = [B(q)^e]_{4\times4}\{q\}^e_{4\times1}. \tag{2.8}$$

For elements $(e1)$ with $r_i = r_k = 0$, $u_i = u_k = 0$ (Fig. 2), taking into account (2.1) we get

$$u = \frac{u_j}{a} r = \frac{q_3}{a} r,$$
$$\varepsilon_\theta = \frac{u}{r} = \frac{u_j}{a} = \frac{q_3}{a}.$$

For elements $(e2)$ with $r_i = 0$, $u_i = 0$ (Fig. 3), combining with (2.1) yields

$$u = (q_5 - q_3) + \frac{q_3}{a} r + \frac{q_3 - q_5}{a} z,$$
$$\varepsilon_\theta = \frac{u}{r} = \left(-1 + \frac{r}{a} + \frac{z}{a}\right) q_3 + \left(1 - \frac{z}{a}\right) q_5.$$

The strain matrix of elements $(e1)$ and $(e2)$ can be written

$$\{\varepsilon\}^{e1} = \begin{bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \tau_{rz} \end{bmatrix} = \begin{bmatrix} & & \partial N_1/\partial r & & & \\ & & \partial N_2/\partial z & & & \\ 0 & 0 & 1/a & 0 & 0 & 0 \\ & & \partial N_1/\partial z + \partial N_2/\partial r & & & \end{bmatrix}_{4\times6} \{q\}^{e1}_{6\times1},$$

$$\{\varepsilon\}^{e2} = \begin{bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \tau_{rz} \end{bmatrix} = \begin{bmatrix} & & \partial N_1/\partial r & & & \\ & & \partial N_2/\partial z & & & \\ 0 & 0 & (-1 + r/a + z/a) & 0 & (1 - z/a) & 0 \\ & & \partial N_1/\partial z + \partial N_2/\partial r & & & \end{bmatrix}_{4\times 6} \{q\}^{e2}_{6\times 1}.$$

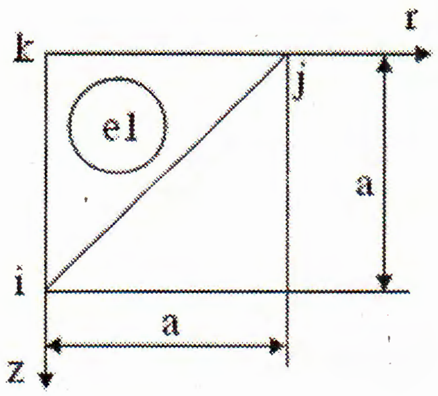

Fig. 2. Elements $(e1)$ with $r_i = r_k = 0$

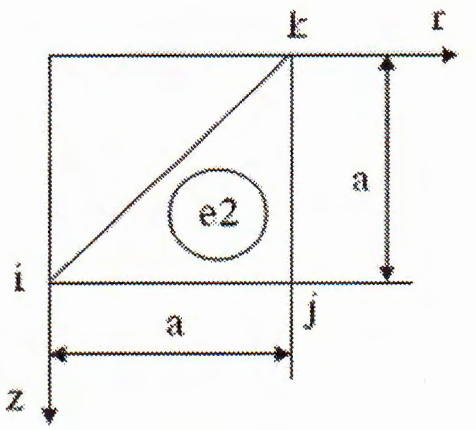

Fig. 3. Elements $(e2)$ with $r_i = 0$

For the infinite element (4) $u_i = 0$ and from (2.6) we get

$$u = q_3 \frac{8R_0}{z^2} r; \quad \varepsilon_\theta = \frac{u}{r} = q_3 \frac{8R_0}{z^2}.$$

The strain matrix of infinite element (4) can be calculated

$$\{\varepsilon\}^{4} = \begin{bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \tau_{rz} \end{bmatrix} = \begin{bmatrix} & & \partial N_1/\partial r & \\ & & \partial N_2/\partial z & \\ 0 & 0 & 8R_0/z^2 & 0 \\ & & \partial N_1/\partial z + \partial N_2/\partial r & \end{bmatrix}_{4\times 4} \{q\}^{4}_{4\times 1}.$$

Elastic Problem

The variation of potential energy of element (e) is written as

$$\begin{aligned} \{\delta U^e\} &= \iiint_{V_e} \{\delta\varepsilon^e\}^T [D] \{\varepsilon\}^e \, dV \\ &= \{\delta q^e\}^T \Big(2\pi \iint_{S_e} [B^e]^T [D][B^e] \, r \, dr \, dz\Big) \{q\}^e, \end{aligned} \tag{2.9}$$

taking herein strain expressions by (2.5), (2.8) and matrix $[D]$ in (1.1).
Denote the stiffness matrix of elements

$$[K^e] = 2\pi \iint_{S_e} [B^e]^T [D][B^e] \, r \, dr \, dz, \tag{2.10}$$

then the relation (2.9) can be written as following

$$\{\delta U^e\} = \{\delta q^e\}^T[K^e]\{q\}^e. \tag{2.11}$$

For infinite elements the stiffness matrix is expressed by formulas

$$[K]_1 = 2\pi \int_{4R_0}^{+\infty} dr \int_{2R_0}^{r} [B]_1^T[D][B]_1\,.r\,dz,$$

$$[K]_2 = 2\pi \int_{4R_0}^{+\infty} dz \int_{2R_0}^{z} [B]_2^T[D][B]_2\,.r\,dr,$$

$$[K]_3 = 2\pi \int_{4R_0}^{+\infty} dr \int_{0}^{2R_0} [B]_3^T[D][B]_3\,.r\,dz,$$

$$[K]_4 = 2\pi \int_{4R_0}^{+\infty} dz \int_{0}^{2R_0} [B]_4^T[D][B]_4\,.r\,dr.$$

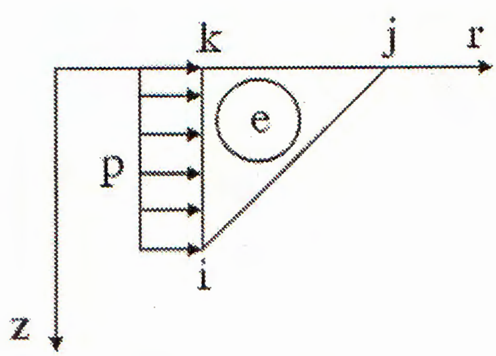

Fig. 4. Element subjected to loading

The variation of work done by external force is calculated as follows

$$\delta A^e = \int_{ik} [\delta u \quad \delta v] \begin{bmatrix} p \\ 0 \end{bmatrix} dl = \delta\{q_e\}^T \int_{ik} [N]^T \begin{bmatrix} p \\ 0 \end{bmatrix} dl = \delta\{q_e\}^T\{P^e\}, \tag{2.12}$$

where the forces matrix of elements is determined

$$\{P^e\}_{6\times 1} = \int_{ik} [N]^T \begin{bmatrix} p \\ 0 \end{bmatrix} dl. \tag{2.13}$$

In the part of the meridian cross-cut half-space with 43 nodes, there are 86 nodal displacement components. Denoting the global vector of displacement $\{q\}$.

$$\{q\}_{86\times 1} = \begin{bmatrix} u_1 & v_1 & u_2 & v_2 & \cdots & u_{43} & v_{43} \end{bmatrix}^T$$

we have relation between nodal and global displacements for element (e)

$$\{q\}^e_{6\times 1} = [L^e]_{6\times 86}\{q\}_{86\times 1}. \tag{2.14}$$

From (2.11), (2.12), (2.14), for half-space we have the variation of potential and work done by external forces.

$$\delta U = \sum_{e=1}^{Le} \delta U^e = \{\delta q\}^T \Big(\sum_{e=1}^{Le}[L^e]^T[K^e][L^e]\Big)\{q\},$$

$$\delta A = \sum_{e=1}^{Le} \delta A^e = \{\delta q\}^T \Big(\sum_{e=1}^{Le}[L^e]^T\{P^e\}\Big). \tag{2.15}$$

where $Le = 62$- is the quantity of elements.
The global stiffness and the forces matrix are determined

$$[K]_{86\times 86} = \sum_{e=1}^{Le}[L^e]^T[K^e][L^e]; \quad \{P\}_{86\times 1} = \sum_{e=1}^{Le}[L^e]^T\{P^e\}. \tag{2.16}$$

According to $\delta U = \delta A$ we derive the equation for finding global displacements in the matrix form

$$[K]_{86\times 86}\{q\}_{86\times 1} = \{P\}_{86\times 1}. \tag{2.17}$$

The strain and stress of elements can be calculated by

$$\begin{aligned} \{\varepsilon_e\} &= [B_e]\{q_e\}, \\ \{\sigma_e\} &= [D]\{\varepsilon_e\}. \end{aligned} \tag{2.18}$$

In an elastic problem the arc-length of the strain trajectory is expessed as

$$s_e = \varepsilon_u = \sqrt{\frac{2}{3}e_{ij}e_{ij}}. \tag{2.19}$$

Plastic Problem.
The loading process is divided into many steps $n = 1, 2, .., N$. At each step a plastic problem can be solved by stepwise-iterative method, so called modified elastic solution method [1, 2]. The results at $(n-1)$-th step are basic data for n-th step.
At n-th step, with iteration k=0, the stress-strain relation can be determined by (1.6)

$$\{\Delta\sigma_e^{(n,0)}\} = \big[D\big(\sigma_{ij}^{(n-1)}, s^{(n-1)}\big)\big]_d \cdot \{\Delta\varepsilon_e^{(n,0)}\}. \tag{2.20}$$

Solving elastic problem with load $\Delta p^{(n)}$ we have displacement increment of nodes $\{\Delta q_e^{(n,0)}\}$, from here the strain-stress increment and arc-length of the strain trajectory can be represented

$$\begin{aligned}
&\{\Delta\varepsilon_e^{(n,0)}\} = [B_e]\{\Delta q_e^{(n,0)}\},\\
&\{\Delta\sigma_e^{(n,0)}\} = \left[D\left(\sigma_{ij}^{(n-1)}, s^{(n-1)}\right)\right]_d \cdot \{\Delta\varepsilon_e^{(n,0)}\},\\
&\Delta s^{(n,0)} = \sqrt{\frac{2}{3}\Delta e_{ij}^{(n,0)}\Delta e_{ij}^{(n,0)}}.
\end{aligned} \tag{2.21}$$

-With k-th iteration ($k > 0$):
We determinate the nodal stress, displacement and arc-length of the strain trajectory

$$\begin{aligned}
&\{\sigma_e^{(n,k)}\} = \{\sigma_e^{(n-1)}\} + \{\Delta\sigma_e^{(n,k-1)}\},\\
&\{q_e^{(n,k)}\} = \{q_e^{(n-1)}\} + \{\Delta q_e^{(n,k-1)}\},\\
&s^{(n,k)} = s^{(n-1)} + \Delta s^{(n,k-1)},
\end{aligned} \tag{2.22}$$

the stress-strain relation can be written

$$\{\Delta\sigma_e^{(n,k)}\} = \left[D\left(\sigma_{ij}^{(n,k-1)}, s^{(n,k-1)}\right)\right]_d \cdot \{\Delta\varepsilon_e^{(n,k)}\}. \tag{2.23}$$

Solving elastic problem with load $\Delta p^{(n)}$ we have $\{\Delta q_e^{(n,k)}\}$. Similary (2.21), we get the values of $\{\Delta\varepsilon_e^{(n,k)}\}, \{\Delta\sigma_e^{(n,k)}\}, \Delta s^{(n,k)}$. The iteration-process is finished when all nodal displacements satisfy the condition

$$\left|\frac{q_i^{(n,k)} - q_i^{(n,k-1)}}{q_i^{(n,k)}}\right| < \delta, \tag{2.24}$$

for a given enough small $\delta > 0$.
Since at each iteration matrix $[D]_d$ in (1.6) changes, thus global stiffness matrix $[K]$ changes accordingly.
If stopping at k-th iteration then we get nodal stress, displacements and arc-length of the strain trajectory at n-th step

$$\begin{aligned}
&\{\sigma_e^{(n)}\} = \{\sigma_e^{(n-1)}\} + \{\Delta\sigma_e^{(n,k)}\},\\
&\{q_e^{(n)}\} = \{q_e^{(n-1)}\} + \{\Delta q_e^{(n,k)}\},\\
&s^{(n)} = s^{(n-1)} + \Delta s^{(n,k)}.
\end{aligned} \tag{2.25}$$

The boundary conditions.
Since a half-space with a hole subjected to axially symmetric loading, thus displacements $u = 0$ at points in symmetric axis (Oz).
The suface load on the lateral suface $r = R_0$ of the hole is distributed as follows

$$\sigma_r = \begin{cases} 0, & 0 \leqslant z \leqslant z_1 \\ -p, & z_1 \leqslant z \leqslant z_2 \end{cases}$$

3. NUMERICAL RESULTS

We consider the half-space with a hole: $R_0 = 0.2\,\text{m}$; $H = 2R_0$, with material constants: elastic modulus $E = 2.10^5\,\text{MPa}$; Poisson's ratio $\nu = 0.34$; yeild limit $\sigma_s = 250$ MPa; $\Phi'(s)/E = 0.2$.

The hole is acted on by begining extenal force $p_0 = 4.1.10^8\,\text{N/m}$ at step $n = 0$, the loading process is divided into 20 steps with increment $\Delta p = 0.3.10^8\,\text{N/m}$. The iteration-process is finished when the condition (2.24) satisfies for $\delta = 0.002$. At step $n = 0$ the body is in the elastic state and at n-th step it is in the elasto-plastic state (with $n > 0$).

In this case after numerical calculation we can see that at element 1 (Fig. 8) the stress intensity gets bigger value than ones at all other elements. Obtained results of the stress intensity and the arc-length of strain trajectory are presented on the Table 1.

Table 1. The stress intensity and the arc-length of strain trajectory at element 1

Step	Loading (N/m)	Number of iteration	$\sigma_u^{max}\,(MPa)$	s^{max}
0	4.1×10^8	0	248.2516	0.00110885
2	4.7×10^8	3	259.3471	0.00136135
4	5.3×10^8	3	272.8651	0.00167050
6	5.9×10^8	2	287.4775	0.00200407
8	6.5×10^8	2	302.4003	0.00234346
10	7.1×10^8	3	317.5545	0.00268715
12	7.7×10^8	3	333.5045	0.00304696
14	8.3×10^8	2	350.1899	0.00342231
16	8.9×10^8	2	367.9128	0.00382034
18	9.5×10^8	2	387.1338	0.00425062
20	10.1×10^8	3	407.6660	0.00471409

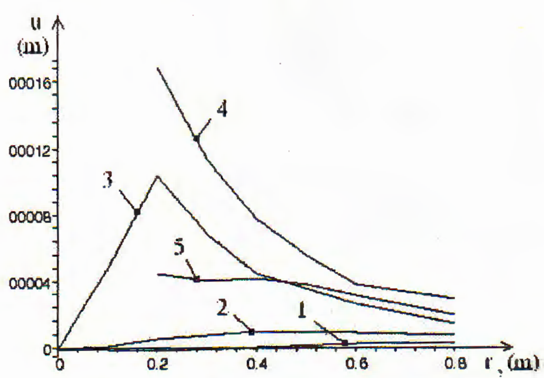

Fig. 5. Graphs of nodals displacements u as a function of r

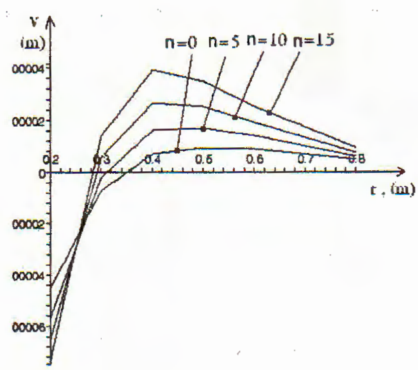

Fig. 6. Graphs of nodals displacements v as a function of r

Graphs of nodals displacements u as a function of r at horizontal lines 1, 2, 3, 4, 5 at step $n = 0$ are shown in Fig. 5 and graphs of nodals displacements v on the horizontal line

5, i.e on the boundary of the half-space with a hole at step $n = 0$, $n = 5$, $n = 10$, $n = 15$ as a function of r - in Fig. 6.
Graphs of the stress intensity of elements (1), (3), (5) as a function of external force p are shown in Fig. 7.

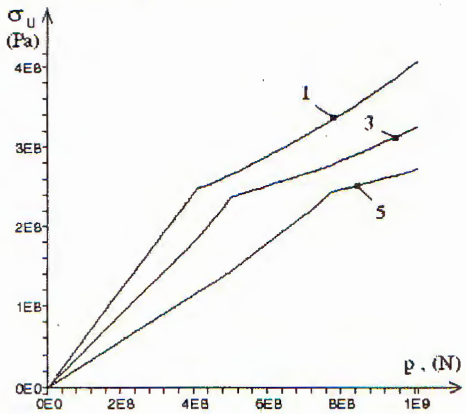

Fig. 7. Graphs of the stress intensity as a function of external force p

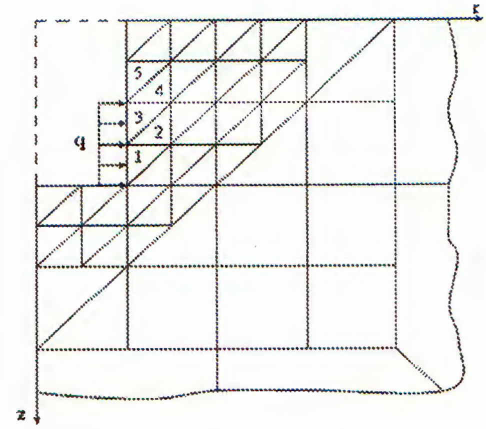

Fig. 8. The plastic domains at step $n = 11$

The plastic domains (1, 2, 3, 4) appear at steps $n = (1, 5, 6, 11)$ respectively (Fig. 8).

The plastic domains of half-space are shown in Fig. 9 and Fig. 10 respectively at step $n = 16$ and at step $n = 20$. From the results it can be seen that:

- When external force increases, the nodals displacements, stress intensity and strain intensity of elements also get increased.
- The plastic domains increases following external force. Plastic domains concentrate near positions subjected to loading.

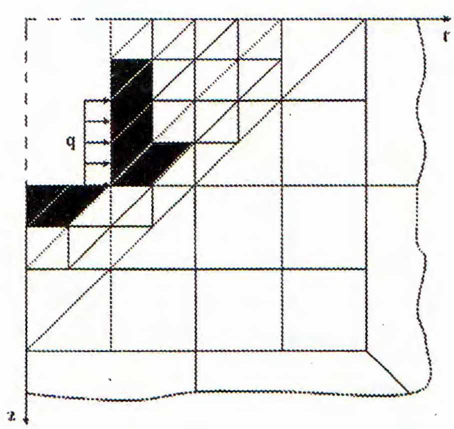

Fig. 9. The plastic domains at step $n = 16$

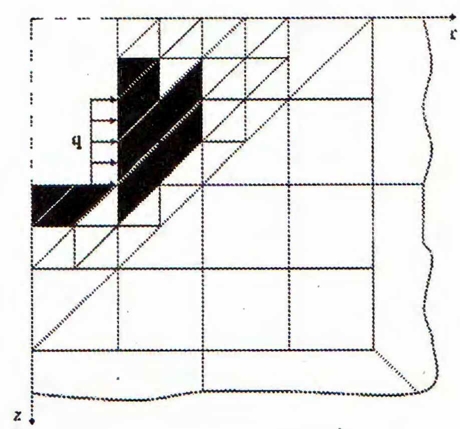

Fig. 10. The plastic domains at step $n = 20$

4. CONCLUSION

- In this paper numerical calculations of strain-stress state of a half-space with a hole subjected to axially symmetric loading by finite element method are presented.

- For solving elasto-plastic problem we have used the modified elastic solution method and relations of the elastoplastic process theory.
- Convergence of iterative process happens quickly after some iterations.

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REFERENCES

1. Dao Huy Bich, Theory of elasto-plastic processes, *Vietnam National University Publishing House*, Hanoi, 1999, (in Vietnamese).
2. Dao Huy Bich, Modified elastic solution method in solving elastoplastic problems of structure components subjected to complex loading, *Vietnam Journal of Mechanics* **22** (3) (2000) 141-156.
3. K. J. Bath and E. L. Wilson, Numerical methods in Finite Element Analysis. *Prentice-Hall*, 1976.
4. V. N. Kukudzhanov and D. N. Shneiderman, Solution of elastoplastic problems of the non-axially symmetric deformation of bodies of revolution, *J. Appl. Maths Mechs* **61** (1) (1997) 503-511.

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BÀI TOÁN ĐÀN DẺO CỦA BÁN KHÔNG GIAN VỚI LỖ TRỤ CHỊU TẢI ĐỐI XỨNG TRỤC

Bài toán đàn dẻo của bán không gian với lỗ trụ chịu tải đối xứng trục được xét đến trong bài toán dựa trên cơ sở lý thuyết quá trình biến dạng đàn dẻo. Lời giải của bài toán đã được tìm ra theo phương pháp Biến thể nghiệm đàn hồi và phương pháp Phần tử hữu hạn. Một vài kết quả giải số được đưa ra đã cho hình ảnh mở rộng miền dẻo trong vật thể cũng như chuyển vị trên biên tự do của bán không gian.